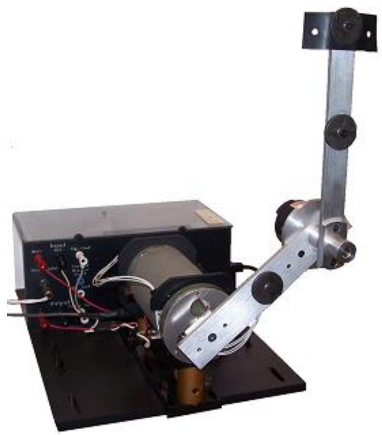
Control of Pendula
a tribute to Mark

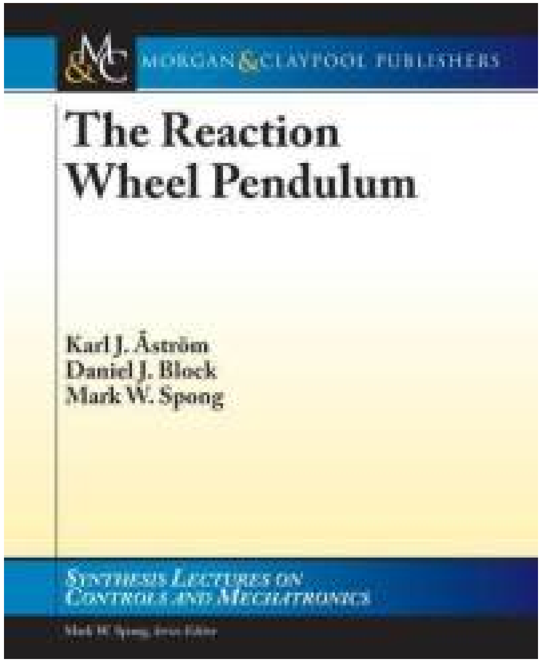
Karl Johan Åström
Department of Automatic Control
Lund University
Sweden

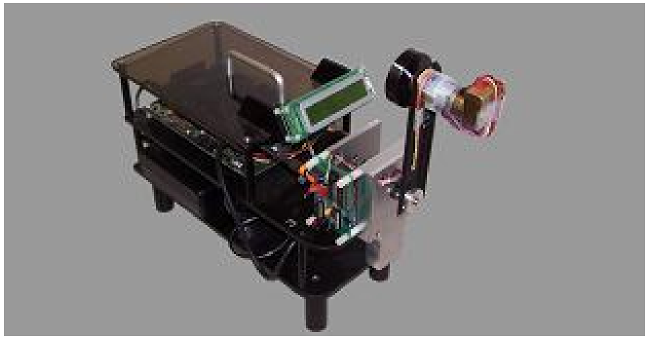
Thanks to Javier Aracil, Dan Block, Sabina Brufani, Carlos Canudas de Wit, Francesco Gordillo, Katsuhisa Furuta, Magnus Gäfvert, Henrik Olsson, Mark Spong, Magnus Wiklund Johan Åkesson

Interactions with Mark

- KJ visiting prof Washington University St Louis 1981
- Mark In Lund: sabbatical 1992 opponent 1996
- UIUC 1991, 1997, 1999
- Control lab development Dan Block UIUC 2000
- From applied mathematics to mechatronics







Pumping-damping strategies for the pendulum

Why are pendula interesting?

- Good prototypes for many control problems.
  - Stabilization of unstable systems
  - Large transitions (*swing-up*)
  - Domain of attraction, global stability
  - Friction compensation
  - Safe manual control

- Graduated difficulties and good lab processes: Pendulum, Pendulum on cart, Furuta pendulum, Spherical pendulum

- Similar to real engineering problems: Segway, rockets power systems, phaselocked loops, Josephson junctions.

Outline

- Introduction
- Stabilization and swing up
- Smooth strategies
- Safe manual control
- Conclusions

Swing-up of Simple Pendulum

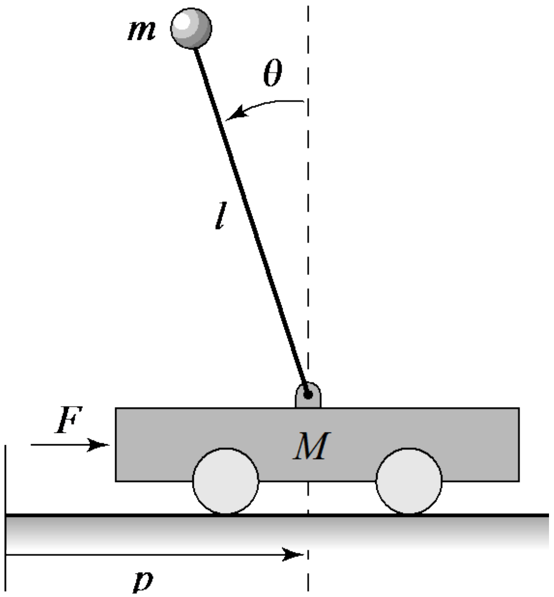
- Simple pendulum

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \sin x_1 + u \cos x_1$$

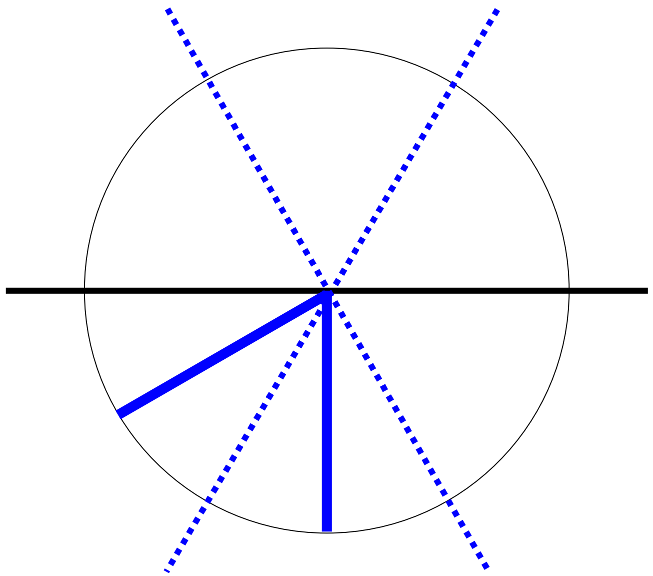
$$E = \cos x_1 - 1 + \frac{1}{2} x_2^2$$

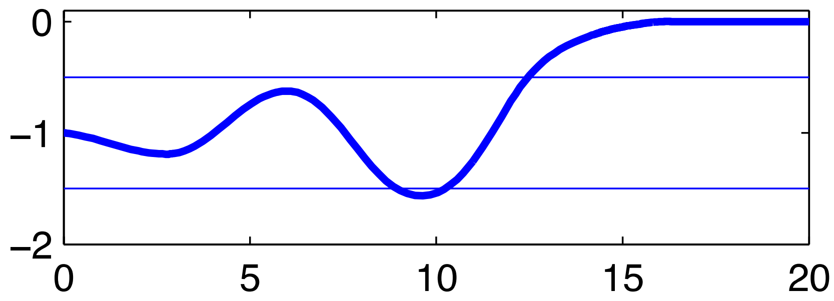
$$\dot{E} = -x_2 \sin x_1 + x_2 \dot{x}_2 = u x_2 \cos x_1$$



- Controlling energy is easy!
- Swing-up energy control + stabilization

Geometric Interpretation



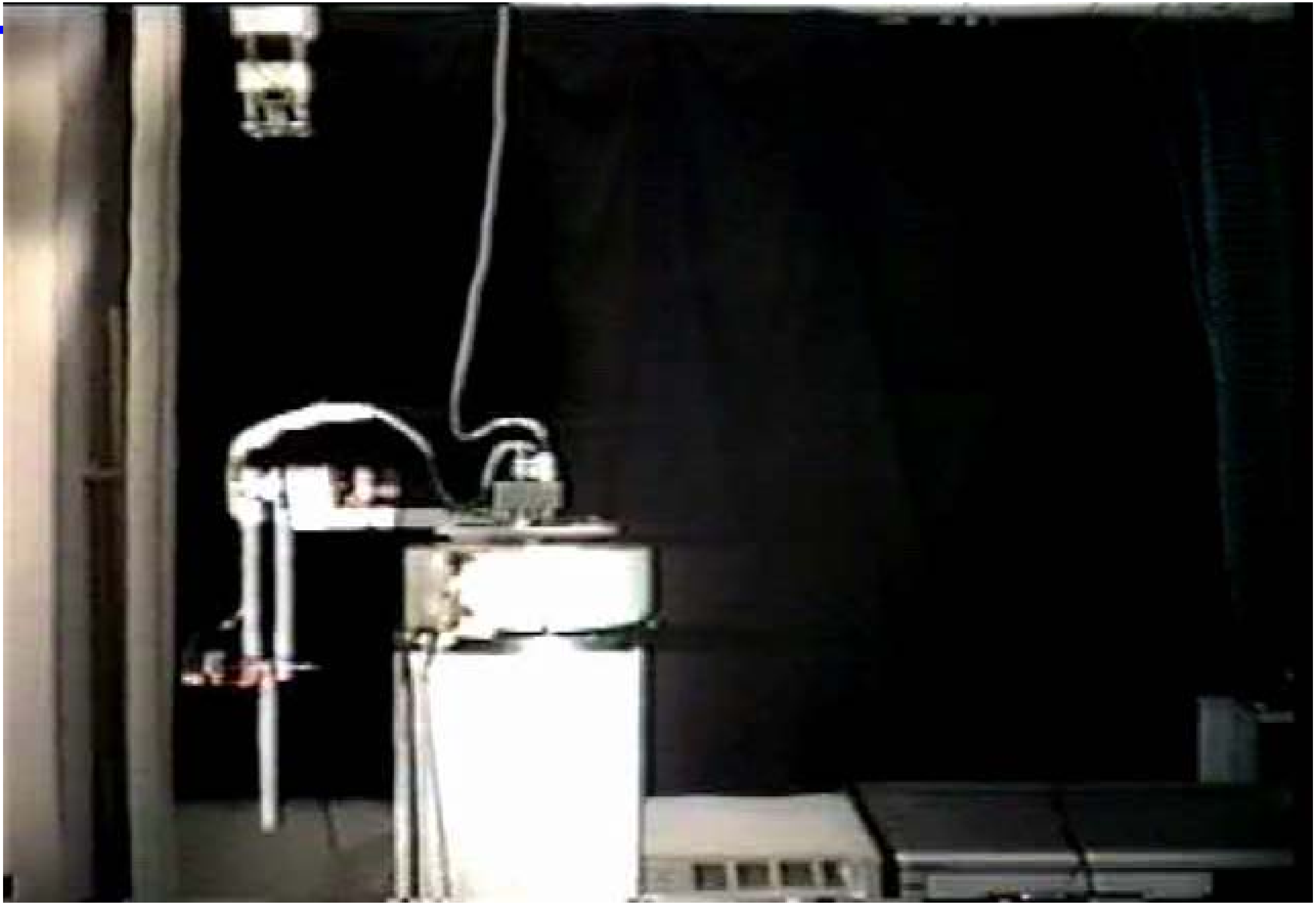


Pivot acceleration: 4g/3, 2g

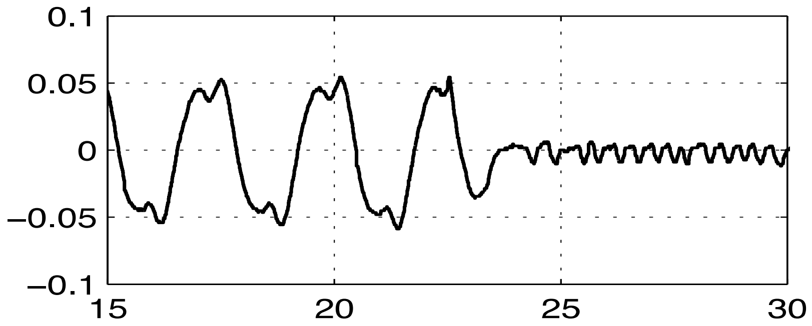
ÅFIH Automatica 2000

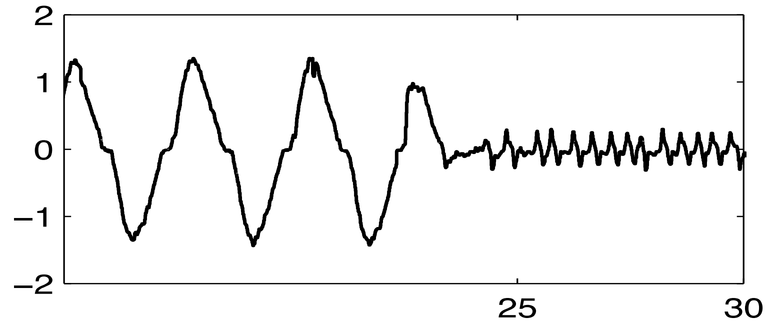
Furuta Pendulum

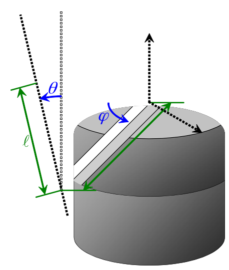
Furuta Pendulum



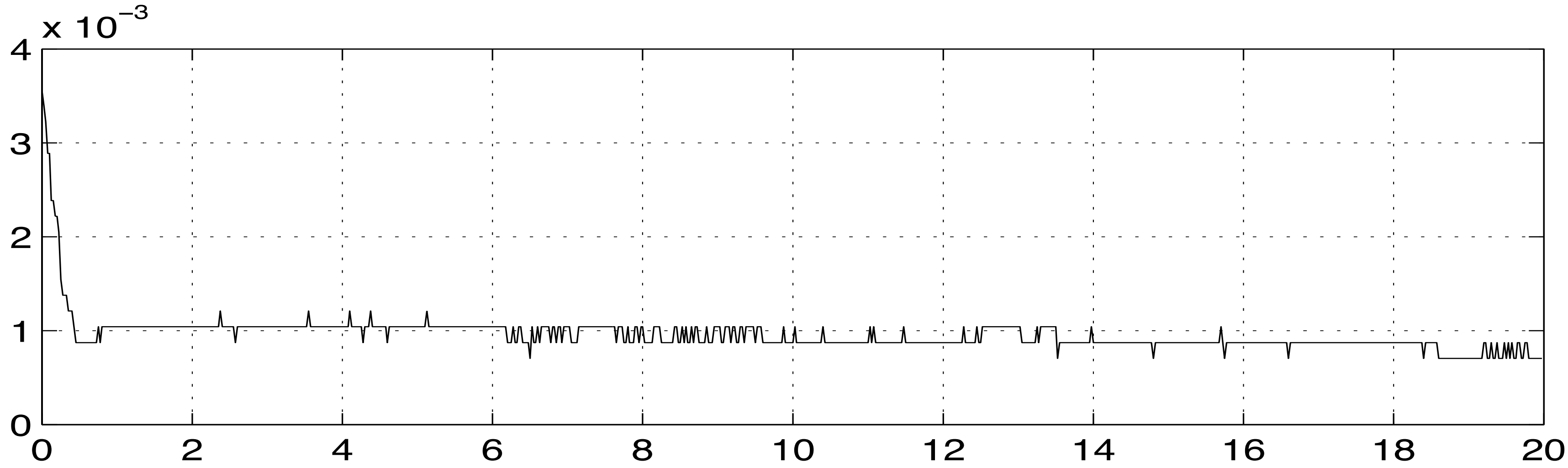
Friction compensation



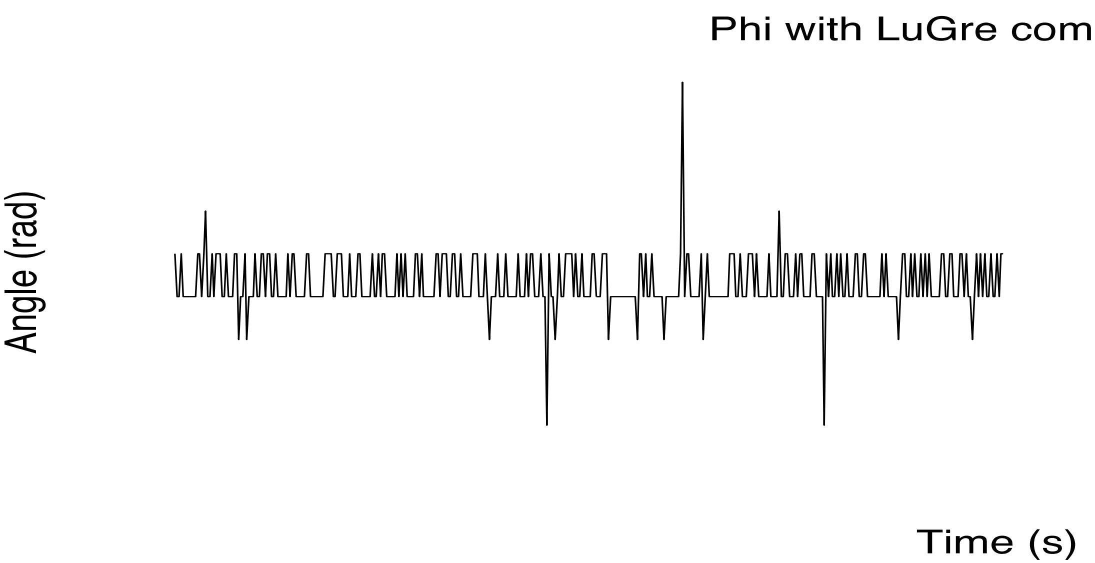




Friction compensation

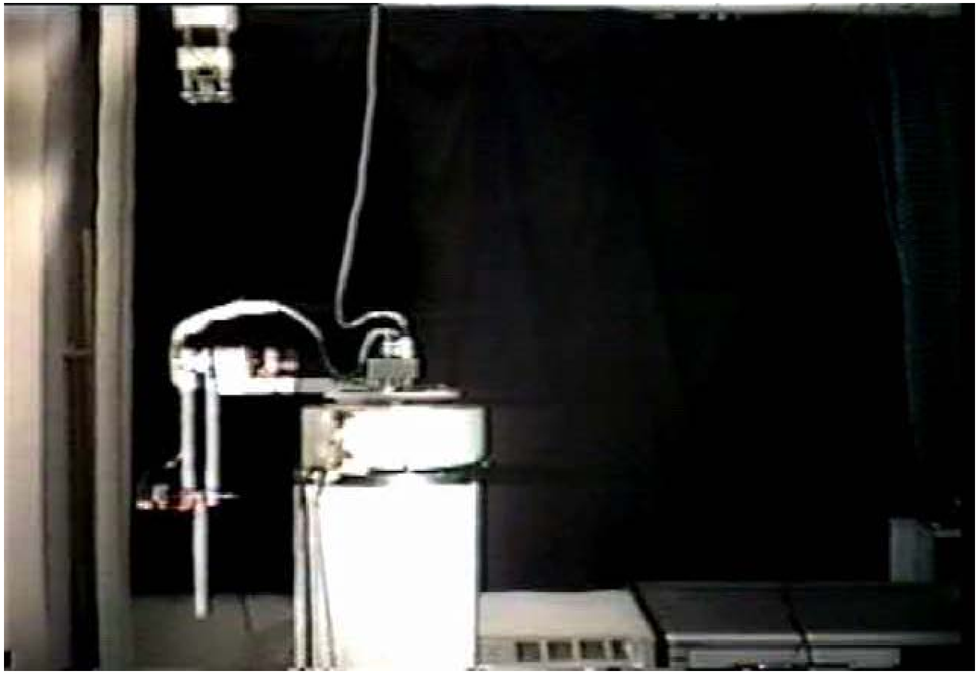


b)



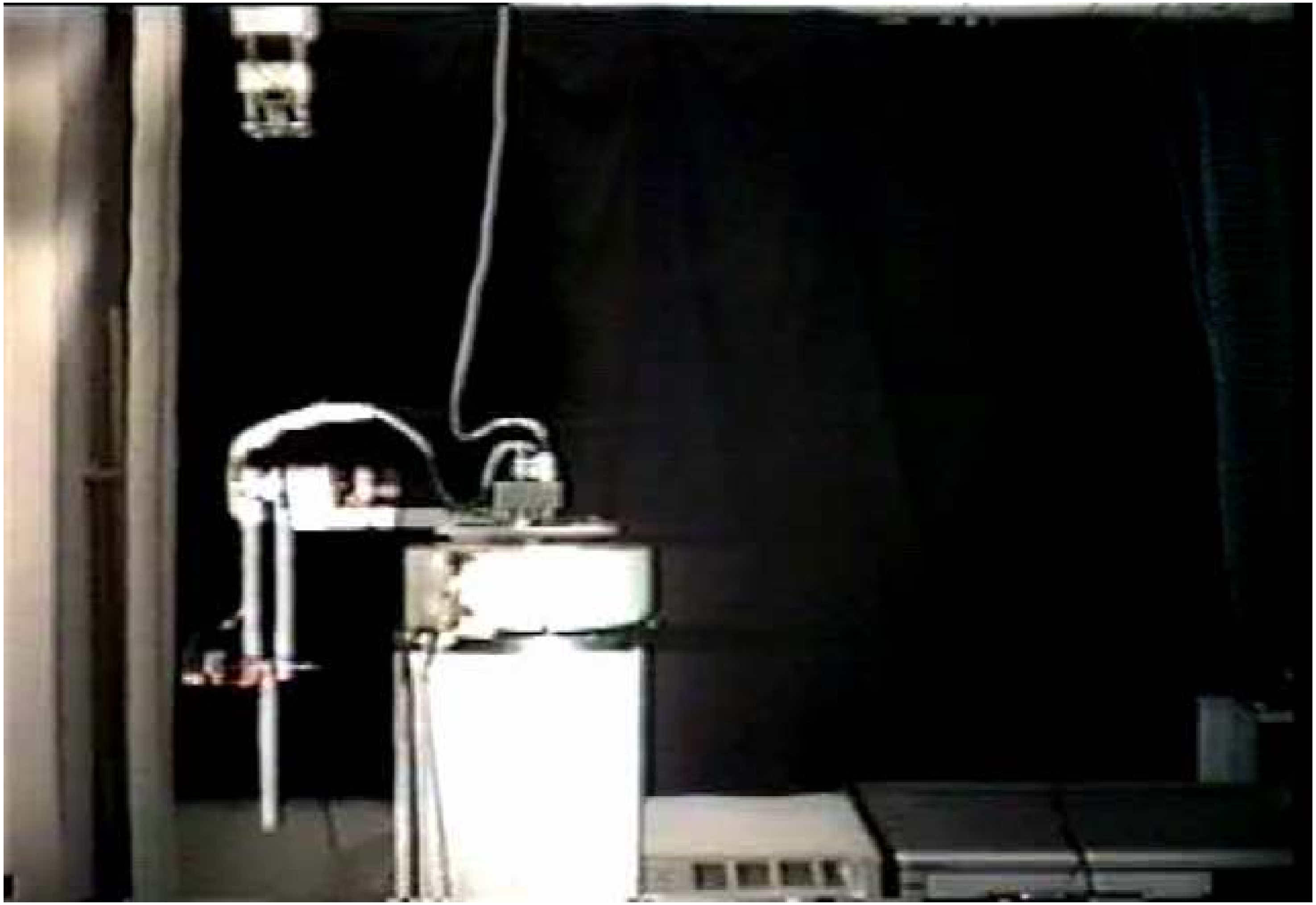
Furuta Pendulum

- Control of chaotic systems
- A double pendulum has chaotic behavior
- Experiments in the Furuta Lab at TIT



ÅFIH IFAC World Congress 1999

Chaotic Furuta Pendulum



ÅFIH IFAC World Congress 1999

Outline

- Introduction
- Stabilization and swing up
- Smooth strategies
- Safe manual control
- Conclusions

Smooth Swing-up

- Shape energy function by feedback

$$\ddot{x} = \sin x + u \cos x$$

- Reverse gravity $u = -2\,\dfrac{\sin x}{\cos x}$
- Choose energy function

$$V(x) = a_0 + a_1 \cos x + a_2 \cos^2 x + a_3 \cos^3 x + \cdot$$

- Simplest case

$$V(x) = 1 - a - \cos x + a \cos^2 x$$

$$V'(x) = \sin x - 2a \sin x \cos x$$

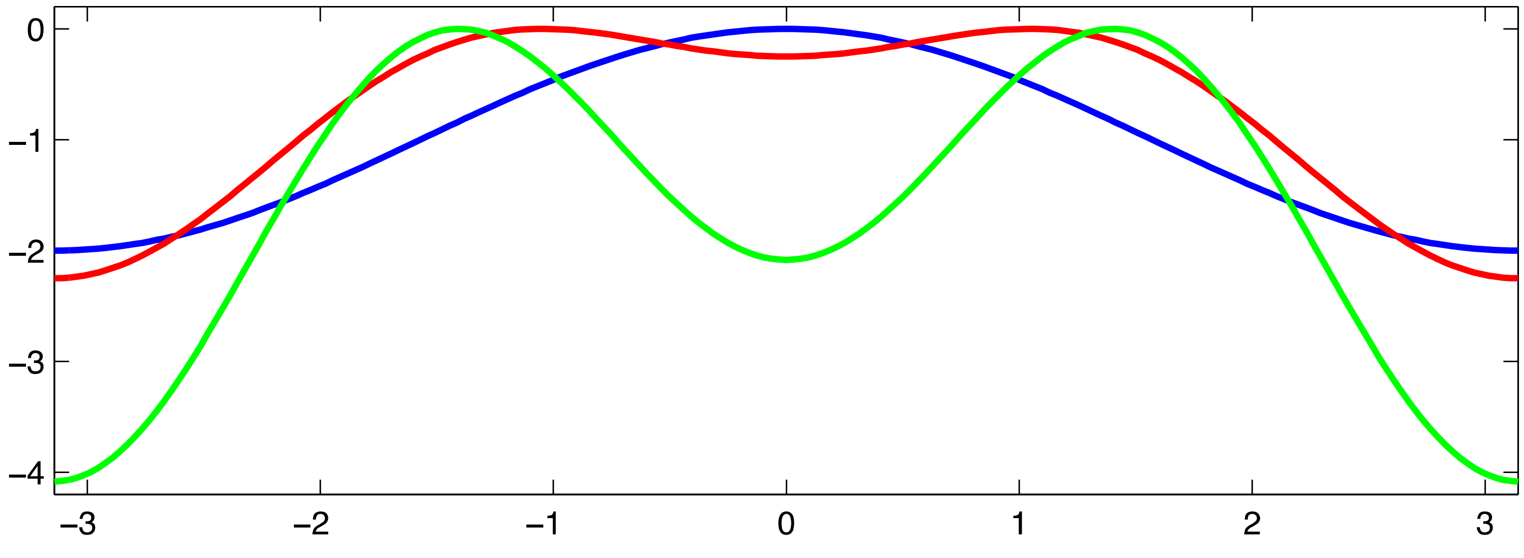
$$u = -2a \sin x$$

ÅAG Automatica 2008

Potential energy with feedback

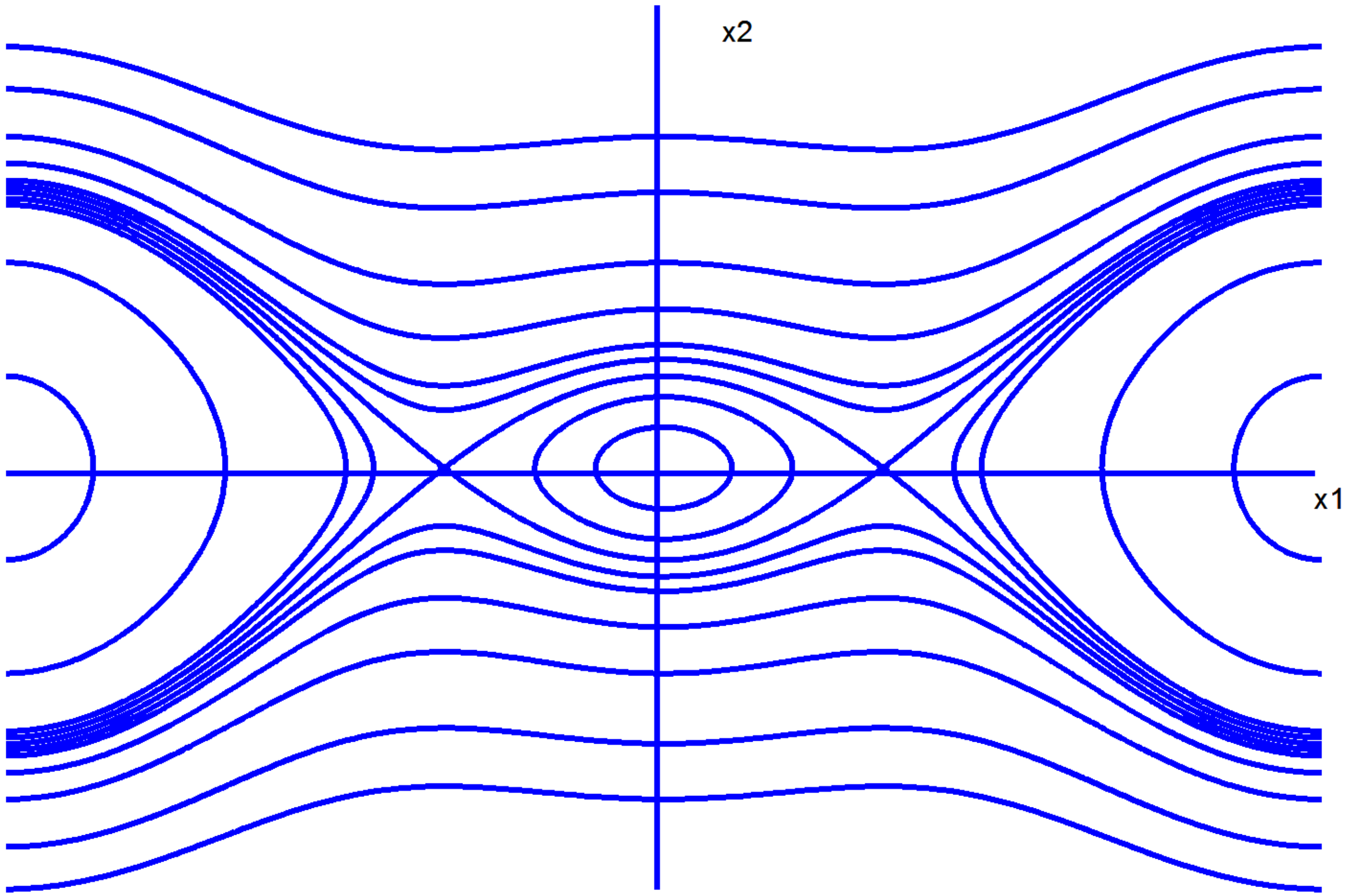
Energy function

$$V(x) = 1 - a - \cos x + a\cos^2 x \approx \frac{x^2}{2}(1-a)$$



Total energy

$$E(x) = 1 - a - \cos x + a\cos^2 x + \frac{1}{2}\dot{x}^2$$



Damping and pumping

$$E(x) = 1 - a - \cos x + a \cos^2 x + \frac{1}{2}\dot{x}^2$$

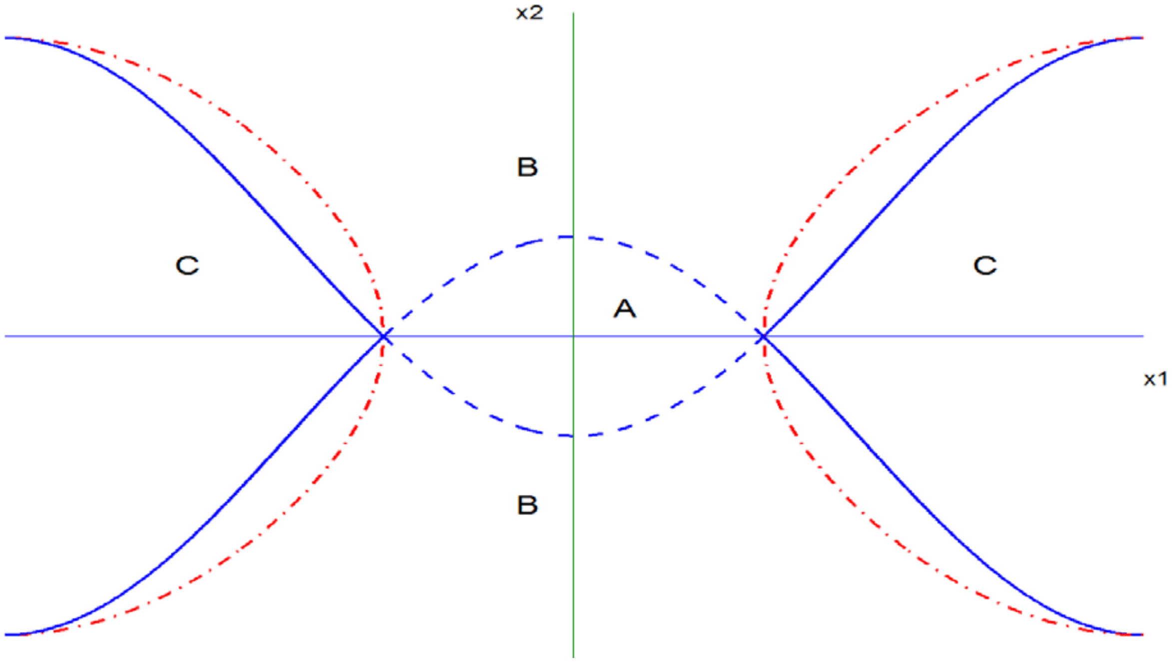
Add damping term to control law

$$u(x_1, x_2) = -2a \sin x_1 - x_2 v(x_1, x_2)$$

$$\begin{aligned}\frac{dE}{dt} &= -x_2^2 v(x_1, x_2) \cos x_1 \\ &= -x_2 F(x_1, x_2) \cos^2 x_1\end{aligned}$$

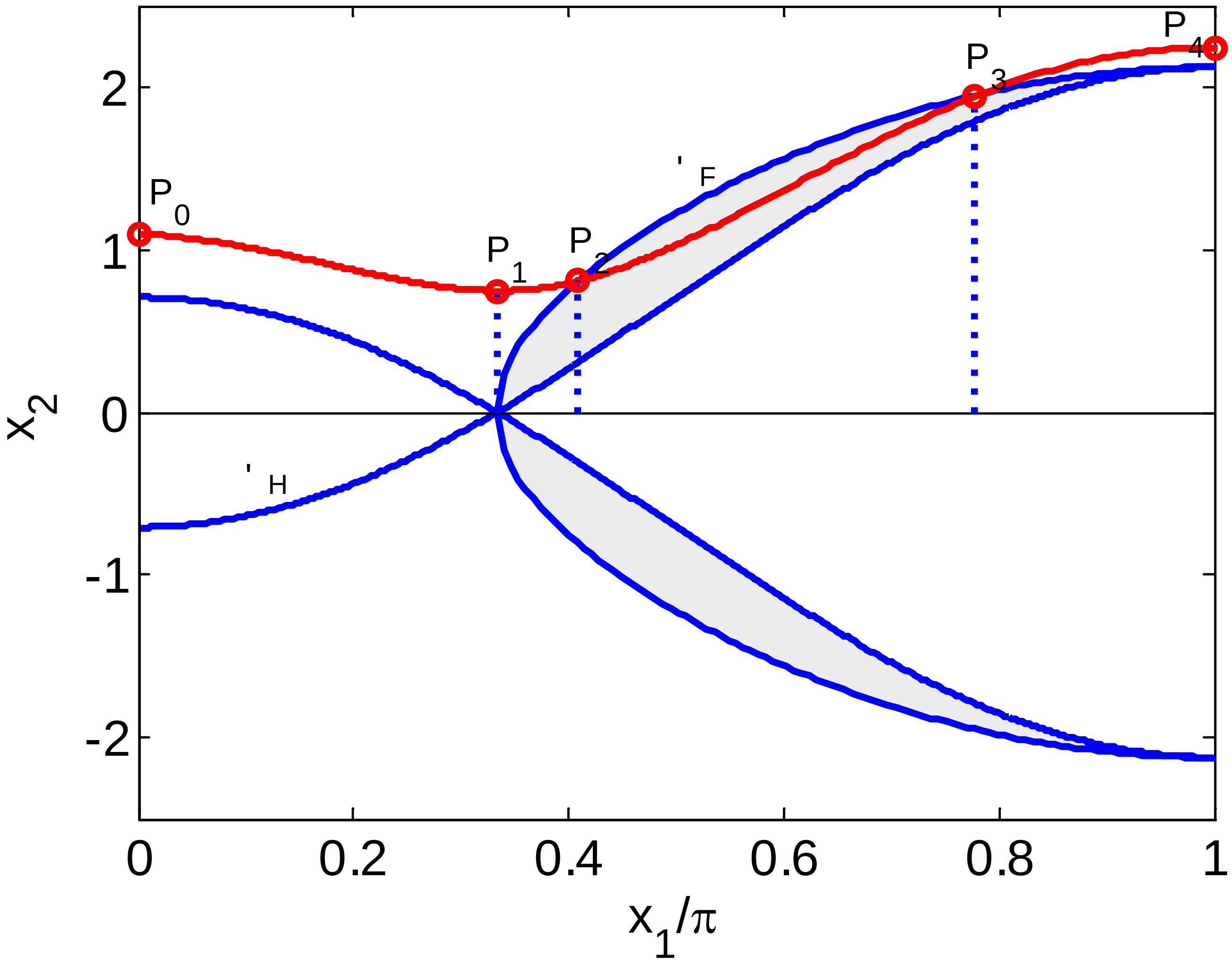
F>0 damping, F<0 pumping

Energy function



$$F(x_1, x_2) = \frac{2a+1}{a}(2a\cos x_1 - 1) + \frac{1}{2}x_2^2$$

Idea of convergence proof



Convergence condition

$$\varphi_H(x) = \sqrt{\frac{1}{2a} + 2a\cos^2 x - 2\cos x}$$

$$\varphi_F(x) = \sqrt{\frac{1+2a}{2a}(1 - 2a\cos x)}, \quad x \geq x_0 = \arccos(1/(2a))$$

$$\Phi(a) = \int_0^{x_0} \varphi_H(x)\cos^2(x) F(x, \varphi_H(x))dx$$
$$+ \int_{x_0}^{\pi} \varphi_F(x)\cos^2(x) F(x, \varphi_H(x))dx$$

Theorem Let *a* be such that Φ(*a*) > 0 and let *b* > 0 then all solutions except those starting at (π , 0) and on the separatrices converge to the origin, a>1 sufficient.

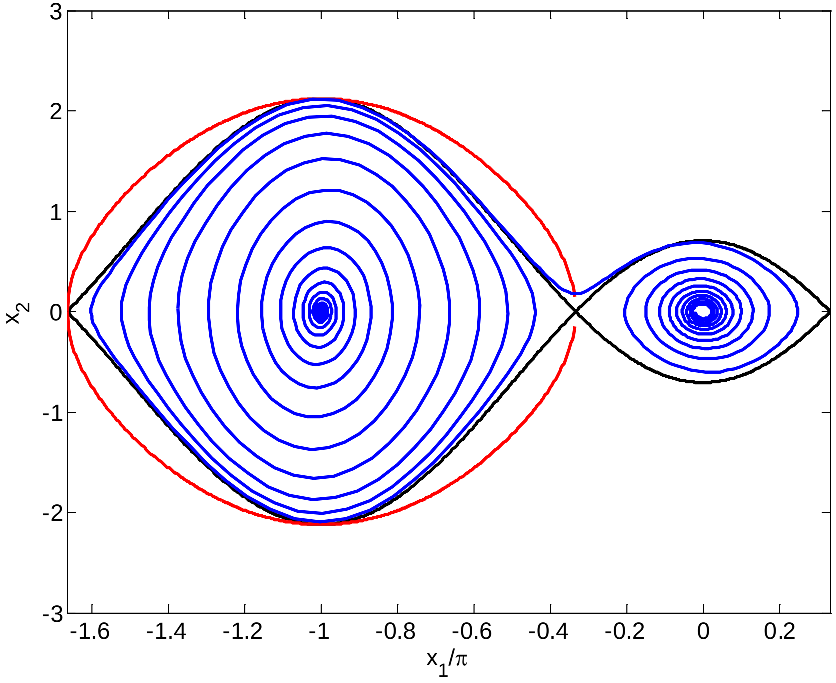
The control law

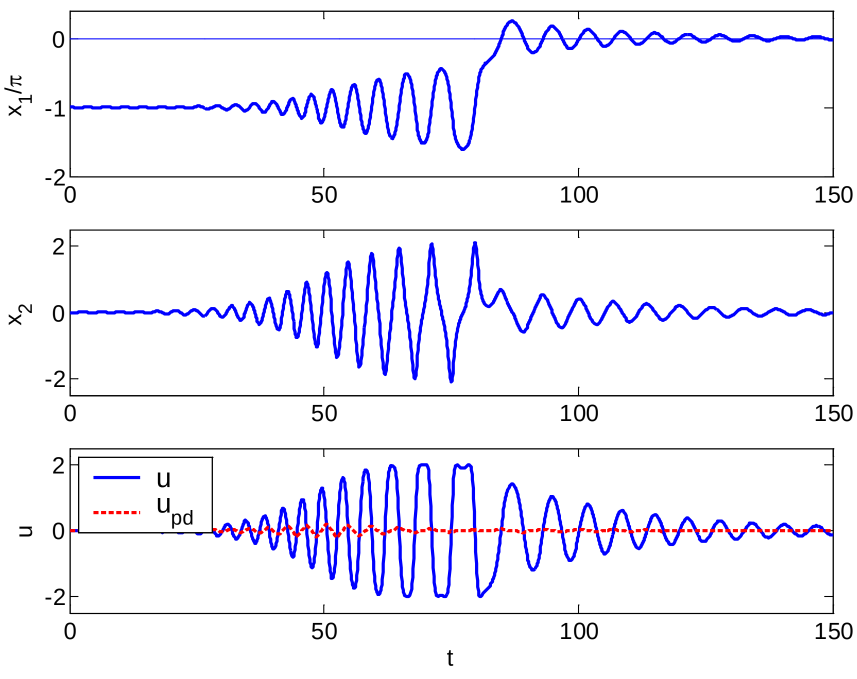
$$u(x_1, x_2) = 2a \sin x_1 + b x_2 F(x_1, x_2) \cos x_1$$

$$F(x_1, x_2) = \frac{2a+1}{a}(2a \cos x_1 - 1) + \frac{1}{2} x_2^2$$

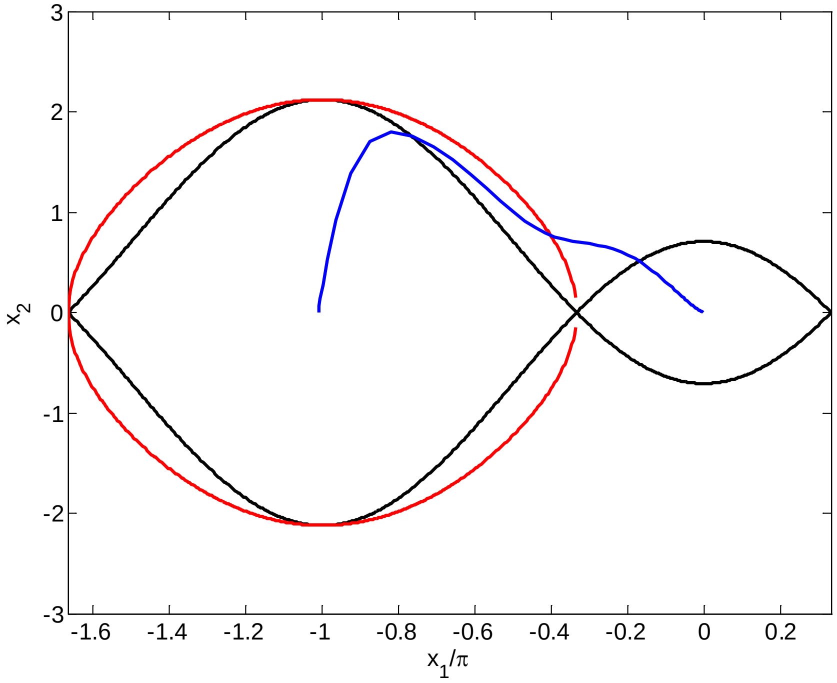
- First term shapes the energy so that the origin is a center
- Second term introduces damping and pumping in appropriate regions
- Parameter a adjusts the width and depth of the potential well
- Parameter b adjusts the rate of damping and pumping

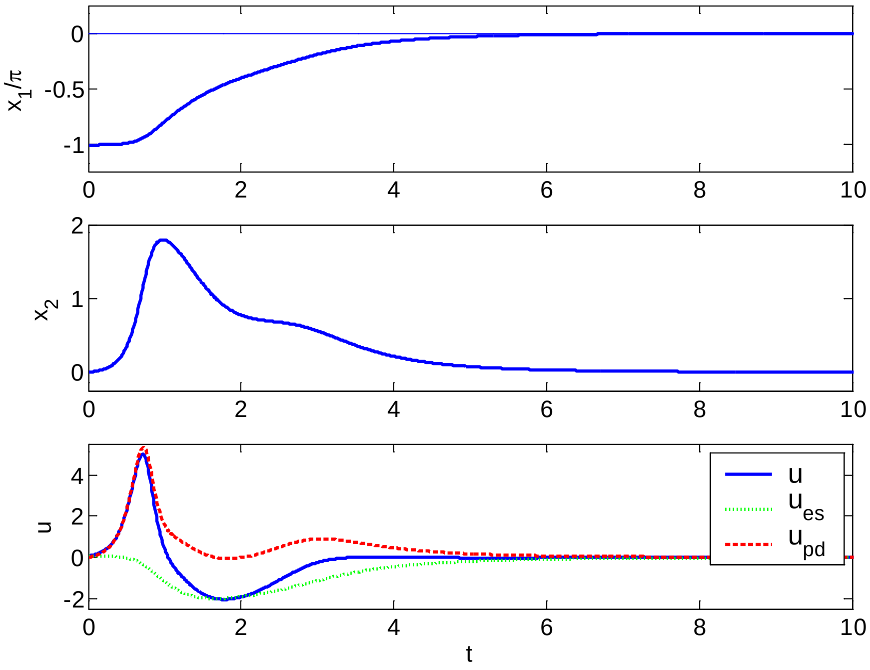
Simulations: a=1, b=0.1





Simulations:a=1 b=3





Summary

- Two simple ideas:
  - Shape potential energy
  - Shape the damping
- The control law:

$$u(x_1, x_2) = -2a \sin x_1 - b x_2 F(x_1, x_2) \cos x_1$$

$$F(x_1, x_2) = \frac{2a+1}{a}(2a \cos x_1 - 1) + \frac{1}{2} x_2^2$$

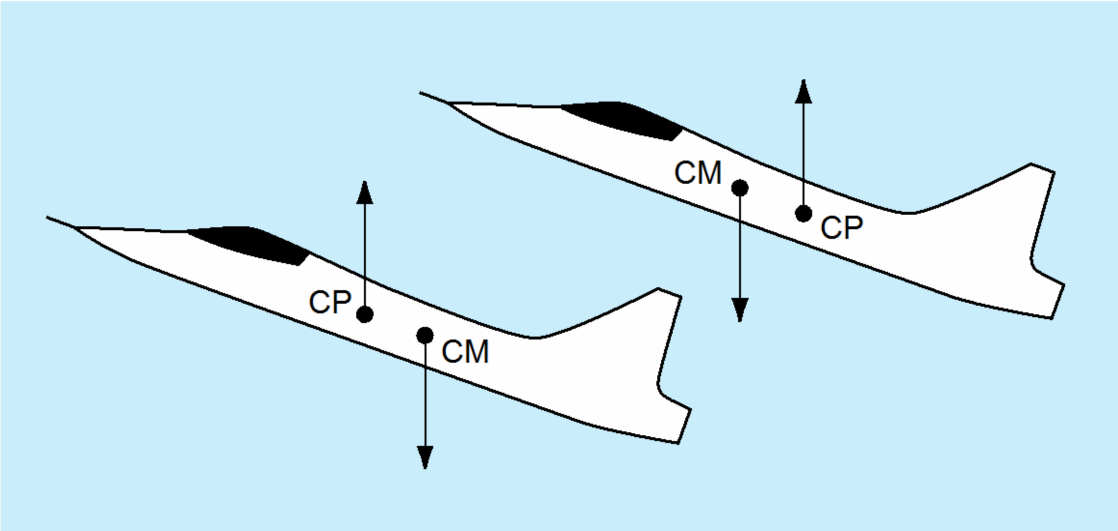
- Parameters a>1 and b have good physical interpretations

Outline

- Introduction
- Stabilization and swing up
- Smooth strategies
- Safe manual control
- Conclusions

Manual Control of Unstable Systems

- High Performance Aircrafts



- Unstable in take-off and landing
- Rate saturations (hydraulics)

JAS Gripen

The JAS Gripen

Equivalent Pendulum Problem

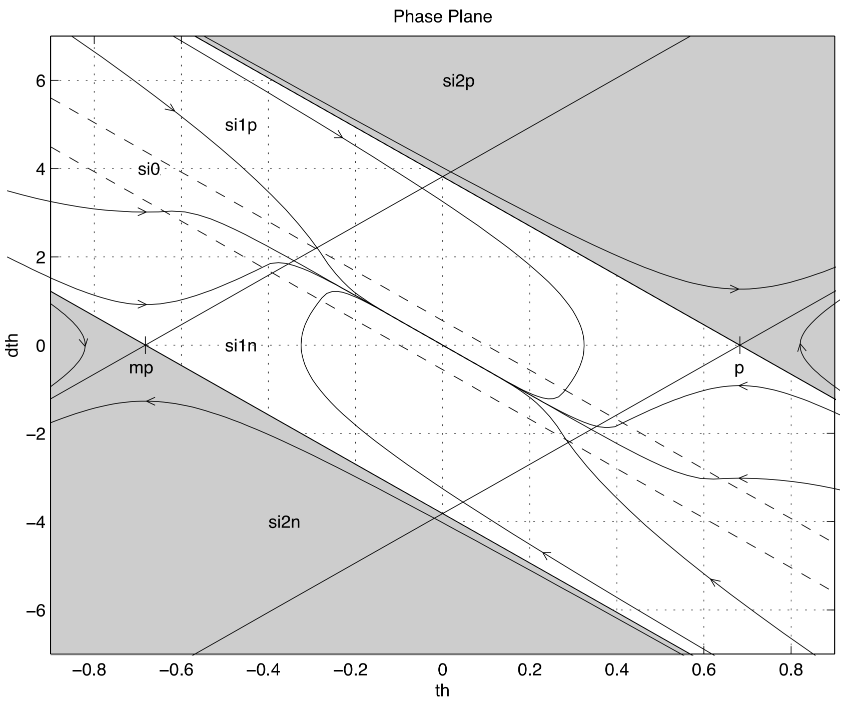
- Stabilize the Furuta pendulum
- Control the arm manually
- Guarantee stability
- Acceleration of pivot limited

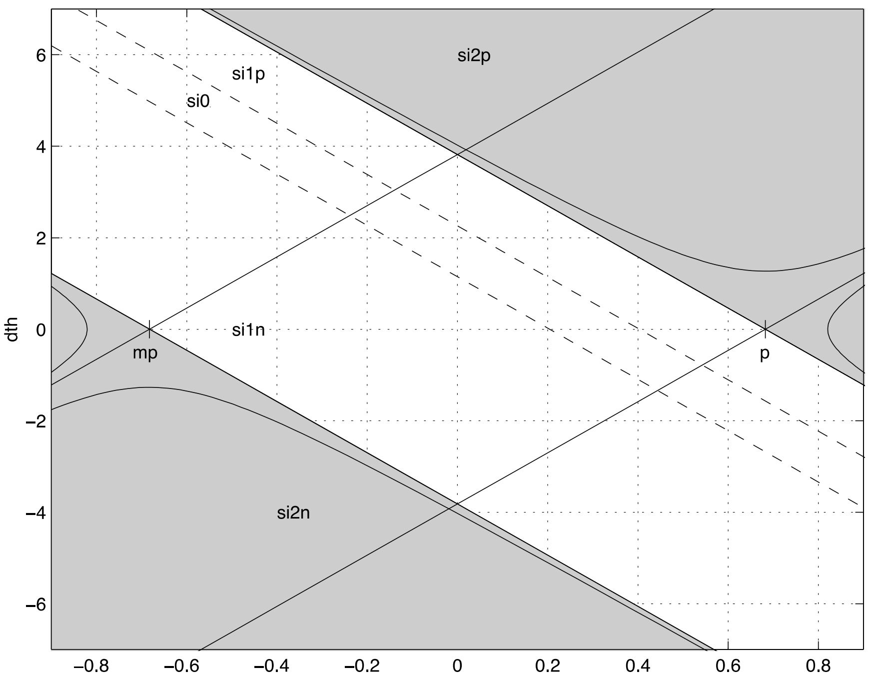
Phase plane - linear case

$$\dot{x}_1 = x_2$$

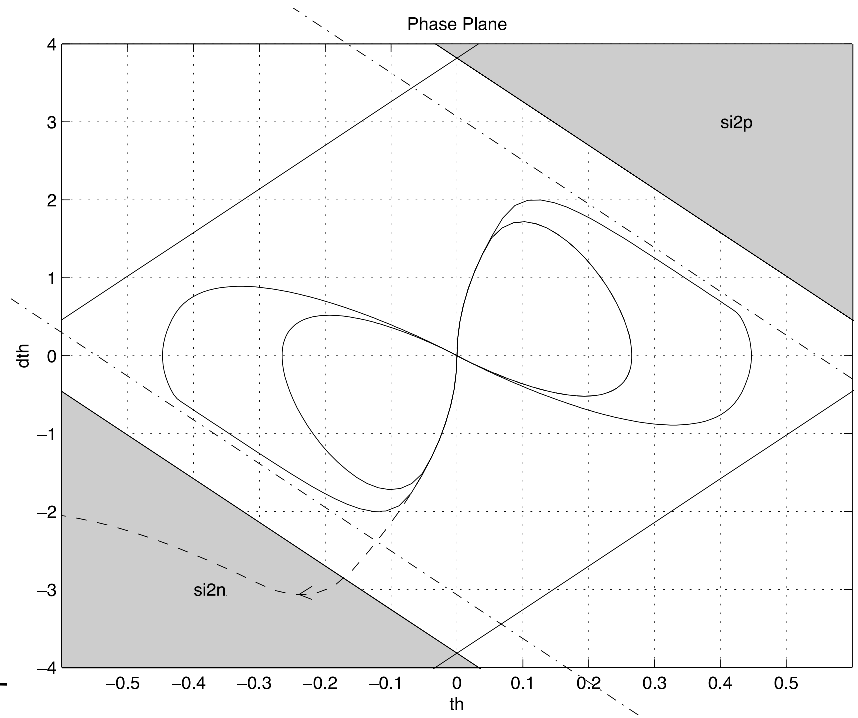
$$\dot{x}_2 = x_1 + u$$

$$u = -\mathrm{sat}_a(k_1 x_1 + k_2 x_2 + \mathrm{sat}_{\mathrm{m}}(\mathrm{u}_{\mathrm{m}}))$$





Restrict authority of manual input



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \sin x_1 + u \cos x_1$$

$$u = -\mathrm{sat}_1(k_1 x_1 + k_2 x_2 + \mathrm{sat}_2(\mathrm{m}))$$

- Assign control authorities to manual control and stabilitzation

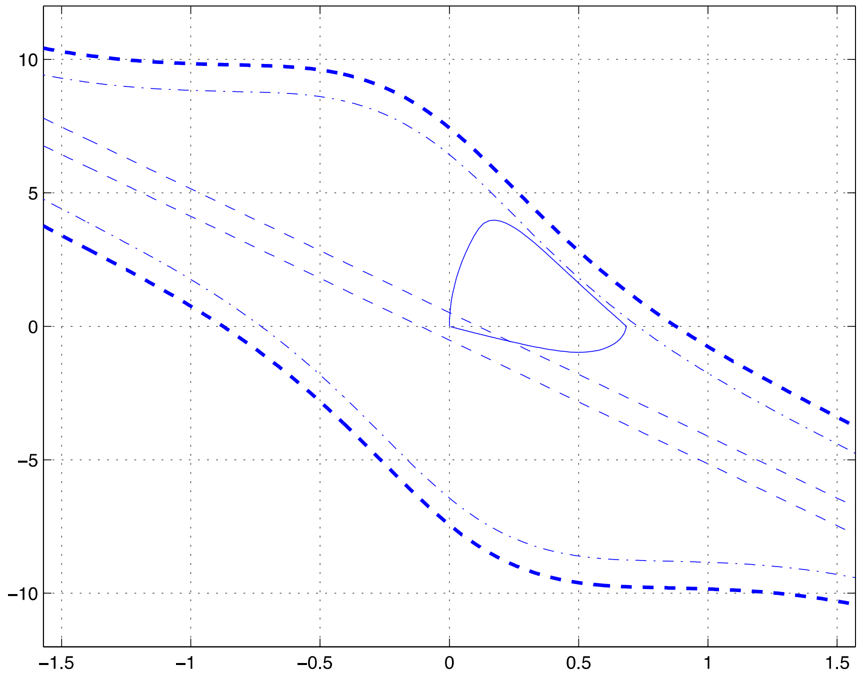
Nonlinear Furuta pendulum

$$\dot{x}_1 = x_2$$

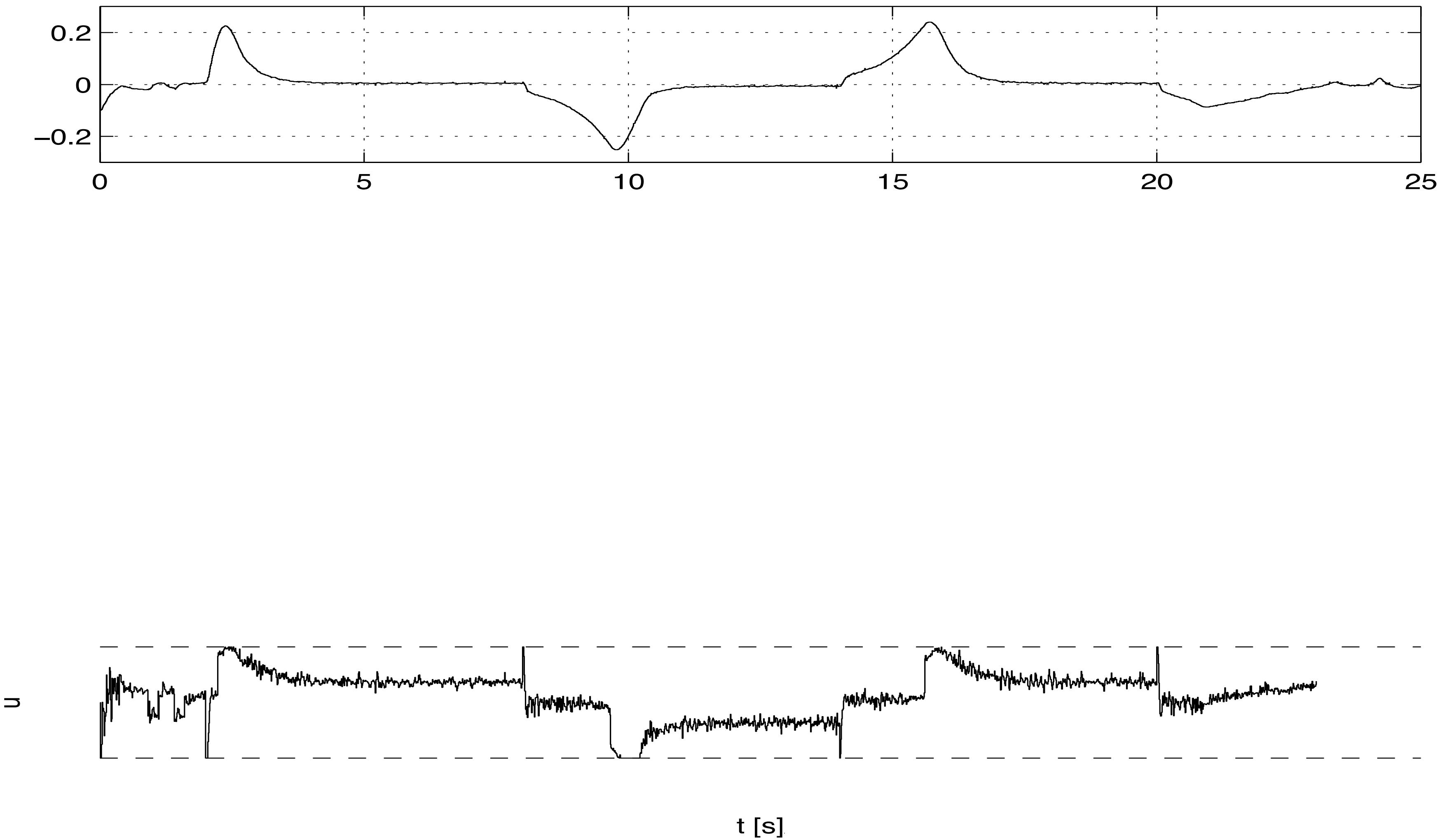
$$\dot{x}_2 = \sin x_1 + a x_3^2 \sin 2x_1 + u \cos x_1$$

$$\dot{x}_3 = u$$

- Scheduling important
- No proof



Experiments



Manual Control Experiment

Outline

- Introduction
- Stabilization and swing up
- Smooth strategies
- Safe manual control
- Conclusions

Summary

- Pendula are good prototypes for many control problems: stabilization, nonlinear behavior, large transitions,
- Interesting theoretical problems
- Experiments are feasible: pendulum on cart, Segway, double Furuta pendulum, Pendubot, reaction wheel pendulum, …
- Mark has made important contributions